Measurement of the Speed of Light in Air

   Measuring the speed of something is a familiar process, just measure the time it takes to travel a given distance. But as Galileo discover, light is much faster that anything else encountered in everyday life. Foucault (1850) and later Michelsen (1878) used a method described below to make some of the first truly accurate measurements of the speed of light. This method involves shining a light from a source on a rotating mirror. The light is reflected by the rotating mirror to a fixed mirror. The fixed mirror reflects the light back to the rotating mirror, which has rotated slightly in the time the light has taken to traveled to the fixed mirror and back. Because the rotating mirror is at a slightly different angle, the light does not return to the source but is displaced slightly. The amount of displacement is related to the rotational frequency of the mirror, and the time it took light to travel from the rotating mirror to the fixed mirror and back. Thus both the distance and the time light takes to travel this distance can be measured. From these measurements the speed of light can be determined.
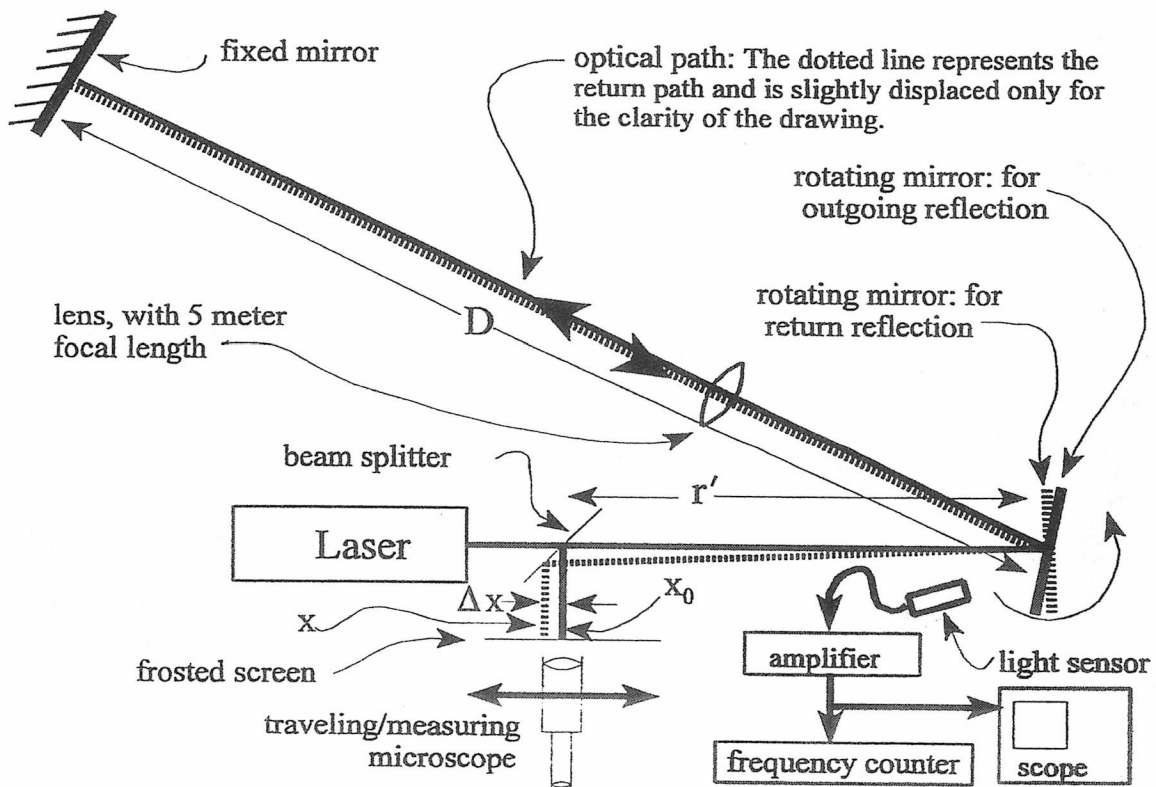

Figure 1   General arrangement of the equipment used to measure the speed of light. For clarity, the return path of the light from the fixed mirror is displaced slightly, and the angle of the rotating mirror for the return reflection is exaggerated. Also the physical optical path is not in plane as shown in this drawing.

D = distance from the rotating mirror to the fixed mirror.
r = distance from rotating mirror to screen = r'+ distance from the beam splitter to the frosted screen.
x_0 = position of the light spot if the mirror did not rotate
x = position of the light spot when the mirror is rotating.
$\Delta x = x - x_0$ displacement of the light spot associated with the rotation of the mirror.

PROCEDURE: Once the optics are setup and all of the relevant optical path lengths are measured, the general procedure is simple enough. For a given rotational frequency of the rotating mirror, one gets a corresponding location of the light spot on the frosted screen. The frequency of rotation and the location of the spot of light, form the basic ordered-pair data point. Measure a series of counter frequency - location of light spot data pairs. Plot the data as you go, extract the slope and convert it to an estimate of the speed of light. Do a least-squares analysis of your data.

Optics Setup: Setting up the optics correctly requires just a bit of care and patients. First note: **To manually rotate the rotating mirror for setup, use the special tool that is attached to the rotating mirror assembly by a bead chain.** This tool extends the axis of rotations when inserted from below and allows one to rotate the mirror.

When aligning optics, one usually works stepwise. For example, if the light must to go from A to B to C to D . . . , one usually aligns the optics to get the light from A to B before working on the section from C to D. Unless large changes are needed, even a rough alignment of the system followed by a careful alignment is a waste of time, because a small change in the beam from A to B may completely shift the beam away from point C.

In this setup there is one exception to this stepwise process, and that is the setting of the lens. Because the lens will steer an off-axis light ray, it is probably better to completely align the optics without the lens and then insert the lens such that it does not cause any deflection of the central ray. Understanding the two functions of the lens and its focal length may help you know the best spot to place it.

Measure the angle of divergence of the laser. Test to see if the position or angle of the beam splitter effects the outgoing beam. If it does, then it must be set before aiming the beam from the laser to the rotating mirror. Once the laser is aim on the center of the rotating mirror, adjust this mirror so that the light is reflected to the fixed mirror, and so forth through the optical system.

Light Sensor Setup: Setting up the light sensor is putting it in the best position to receive the maximum intensity of the reflected light. One can get it close just by eye. With the laser on, the mirror rotating, and the light shield on the sensor slid back, one can see the clear plastic around the sensor is aglow with laser light, when the sensor is in a good position. The final method of optimizing the position of the sensor is to optimize its output signal. This can be done by putting the signal into an oscilloscope. Record the signal heights and make a sketch of the signal and record the relevant scope settings. While the scope is connected, it is suggested that you manually rotate the mirror to see how the signal varies with the angle of the mirror (set scope to auto trigger for this).

With the signal going to both the oscilloscope and the frequency counter, use the scope to estimate the frequency and compare it with the frequency read from the frequency counter. Which do you think is more accurate? Why? Record the switch and dial settings on the frequency counter.

Measuring Lengths   Measure the lengths of the various parts of the optic path. From the error analysis equation, Eq. (3), you should determine the required precision of these measurements.

Traveling/measuring microscope   It may be easier to focus on something more distinct than the light spot on the frosted screen when initially positioning and focusing the microscope.

Taking Data You should try to take a number of data pairs (position vs. frequency) for as large of a range of displacement (or frequency) as possible. WHY? You also need to estimate the uncertainty in these measurements. The uncertainties in these measurements are not simply the uncertainties in the reading of the frequency counter or the dial-indicator distance gauge on the traveling microscope. So to see this, fix the position of the telescope and then adjust the rotational frequency until the spot of light is centered, and record this frequency. Next misadjust the frequency and then re-center the spot. The frequency is probably a bit different from before. The uncertainty in your readings can be estimated by taking multiple measurements. The mean or average of several frequency measurements, f_1, f_2, f_3, ... f_n is

$$<f> = \frac{1}{n}\sum_1^n f_n \text{, and the associated uncertainty is } \sigma_f \cong = s_f = \sqrt{\frac{1}{n-1}\sum (f_n - <f>)^2} \text{ .}$$

Consider if it is easier to take multiple position measurements at a given frequency or multiple frequency measurements for a given displacement.

READING:

Data Reduction and Error Analysis for the Physical Sciences, 2nd Edition by P.R.Bevington and D.K.Robinson,

      Introduction to error analysis and statistics,: p 1-15,

      Good general comments about statistics: p.57,58,

      Significant figures: p. 4,14-15 (rules on pages 14-15),

      Averages and uncertainty of an average,: p. 53-57, 71 (equations on page 71),

      Error propagation: p. 41-50 (equations on page 50),

      Linear least squares analysis: p. 96-114 (equations with errors pages113,114 and for the special case where all uncertainties are equal is page 104 halfway down).

OR

Experiments in Modern Physics, by A.C.Melissinos,

      Error Propagation: p.467-473,

      Averages and uncertainty of an average,: p. 446-447, and

      Least squares analysis: p 462-463.

QUESTIONS

Q1.   What is the accepted speed of light and its associated uncertainty? What method was used in the most recent measurements?

Concerning the setup of the experiment.

Q2.   What is the angle of divergence of the laser?

Q3.   How does the position and angle of the beam splitters affect position or angle outgoing beam?

Q4.   Why is a frosted screen used for viewing rather than simply viewing the spot of light directly with the traveling microscope?

Q5.   Between the rotating mirror and the fixed mirror, the outgoing and the incoming light wave occupying the same space. Must we consider the light in this region to be a superposition of these two waves and worry about interference patterns? Why?

When the optics are properly aligned, the lens has two roles. To gain some understanding of the role of the lens in this experiment, note that the lens has a focal length of 5 meters, and note the positions of various optical elements in (integer) units of this focal length.

Q6.   Sketch the path of the light diverging from the laser, through the lens, to the fixed mirror?

Where is the light from the laser focused? Where is the reflected light from the fixed mirror focused? Why can the rotating mirror be neglected when considering this path?

Q7. The rotating mirror acts as a source of diverging light, because as it rotates, its reflected light sweeps out a large triangular chunk of a plane. Sketch and describe what effect the lens has on this diverging light.

Q8. What is the effect of the lens when the light does not pass through the center of the lens? Consider the light from the laser to the fixed mirror with the outgoing beam passing through the right edge of the lens. What is the effect on the light if the light is not parallel to the axis of the lens?

Q9. How does the angle of the beam splitter relative to the direction of the outgoing laser light alter what is seen on the screen?

Q10. How does the angle of the screen relative to the direction of the outgoing laser light affect what is seen on the screen?

Q11. How does the angle of the direction of motion of the traveling/measuring microscope relative to the direction of the outgoing laser light affect the measurement of positions of the spot of light on the screen?

Q12. Describe the signal from the light sensor. How do the room lights, and the sunlight affect its signal?

Q13. What is the function of the amplifier in the circuit associated with the light sensor?

Q14. How does changing the setting of the frequency counter affect its readings and its precision?

Q15. What is the time-averaged fraction of the original laser light that actually strikes the viewing screen? To the light sensor?

Concerning data analysis

Q16. What quantities did you measure and how did you determine the uncertainties in these measurements?

Q17. What is the dominant source of uncertainty? Can anything be done to reduce this uncertainty? Make concrete suggestions.

Q18. Compare these two methods of data analysis:
      1. Use Eq (1) to calculate the speed of light for every displacement measured and then average these values to obtain the final result.
      2. Use Eq. (2) and the slope ($\Delta f / \Delta x$) extracted manually from a graph to determine the speed of light.
      3. Use Eq. (2) in a linear least-squares analysis of the data.
What are the advantages and disadvantages of each of these methods? Can all of three methods be used in this experiment? Do any require extra measurements? How does one handle uncertainties in each case? For example what is the uncertainty of a slope extracted manually?

Q19. How can data taken after slight shifts of the equipment (data from different days) be combined?

DERIVE THE WORKING EQUATION:

> Be aware that you can get the right working equation but derive it incorrectly, because there are several factors of 2 and ½ in derivation.

The working equation is for a single data point could be:

$$c = 4 \pi r D \frac{f}{x - x_0} , \qquad (1)$$

But if there are a number of data points, as one has in this experiment, it is better to graph the data and extract the speed of light from the slope (or equivalently do a least square fit)

$$f = \frac{c}{4 \pi r D} (x - x_0) , \ or \ f = A + \frac{c}{4 \pi r D} (x) \qquad (2)$$

Or

$$x = B + \frac{4 \pi r D}{c} f , \qquad (3)$$

where

c is the speed of light,
r is the optical path length from the rotating mirror to the screen,.
D is the distance from the rotating mirror to the fixed mirror,
f is the frequency of rotation for the mirror,
x_0 is the position of the light spot if the rotating mirror is not rotating,
x is the observed position of the light spot, and
A, B are constants.

In a working equation, the desired quantity is expressed in terms of measurable quantitiesas is the case for Eq(1), Eq(2) and Eq (3). Eq (1) is good for estimating c from a single measurement of frequency and displacement, Δx. Usually the second or third forms of the working equation, Eq (2), and Eq(3) with the desired quantity (in this case, c) related to the slope of a line is more useful when multiple measurements are made. What are the advantages and disadvantages between the two forms of Eq(2) and Eq(3)?

To derive the working equation start with the simple kinematic equation $\Delta x = v \Delta t$ and apply it to this situation. In this experiment, what is the distance, Δx, in terms of measurable quantities? Now, all that is needed is the time interval Δt.

Make a detailed drawing of the region near the rotating mirror, showing the mirror at two angles, at one angle it should be reflecting the light from the laser to the fixed mirror and at the other angle it will be reflecting light from the fixed mirror to the beam splitter.
- Relate time the light travels from the rotating mirror to the fixed mirror and back to the displacement of the spot of light and the angular frequency ω .
- Through what angle has the mirror rotated in this time?
- What is the angle between the light travel from the laser to the rotating mirror and the light traveling from the rotating mirror to the beam splitter?
- What is the displacement of returned light on the frosted screen in term of this angle?

What is the relationship between the angular frequency, ω, of the rotating mirror and the frequency, f measured by the frequency counter? How many pulses per revolution are produced by the light sensor each time the mirror complete one revolution? Why? Test this experimentally if needed.

Putting all of these steps together should result in the working equation. Of course you need to justify any assumptions or approximations used in your derivation.

DERIVE THE ERROR ANALYSIS EQUATION

Derive the error analysis equation associated with Eq (2 and 3), that is:

$$\sigma_c = c_{measured} \sqrt{\left(\frac{\sigma_D}{D}\right)^2 + \left(\frac{\sigma_r}{r}\right)^2 + \left(\frac{\sigma_{slope}}{slope}\right)^2} \quad , \tag{4}$$

where σ_c is the uncertainty in the value of the speed of light,

σ_D is the uncertainty in the distance from the rotating mirror to the fixed mirror,

σ_r is the uncertainty in the optical path length from the rotating mirror to the screen, and

σ_{SLOPE} is the uncertainty in the value of the slope ($\Delta f/\Delta x$)

Note, an uncertainty in a quantity x is also written as δx. Start with general form of propagation of errors (assuming uncorrelated errors) for a function u = f(w, x, y, z) is:

$$\sigma_u^2 = \sigma_w^2 \left(\frac{\partial u}{\partial w}\right)^2 + \sigma_x^2 \left(\frac{\partial u}{\partial x}\right)^2 + \sigma_y^2 \left(\frac{\partial u}{\partial y}\right)^2 + \sigma_z^2 \left(\frac{\partial u}{\partial y}\right)^2 . \tag{5}$$

When the function is a product of powers ($u = w^a x^b y^c z^d$) show Eq (3) reduces to:

$$\left(\frac{\sigma_u}{u}\right)^2 = \left(a\frac{\sigma_w}{w}\right)^2 + \left(b\frac{\sigma_x}{x}\right)^2 + \left(c\frac{\sigma_y}{y}\right)^2 + \left(d\frac{\sigma_z}{z}\right)^2 . \tag{6}$$

For a more detailed discussion of propagation of errors see *Data Reduction and Error Analysis for the Physical Sciences* by P.R.Bevington and D.K.Robinson, p. 41-50; or *Experiments in Modern Physics*, by A.C.Melissinos, p.467-473.

Even if one uses Eq (2) to determine the value of c, Eq (3) provides information concerning the relative importance of various contributions to the overall uncertainty.

THINGS TO REMEMBER TO DO IN YOUR LAB NOTEBOOK
Schematic drawing(s) of the experimental setup
Equipment list
Settings of equipment controls
Significant figures
Full size plot of your data, with errorbars
Include references
Derivation of the working equation
Error analysis